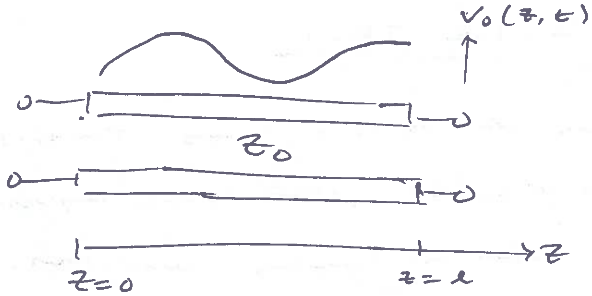
Transmission Line Propagation

Transmission lines are physical lengths of transmission structures that allow TEM waves to propagate through.

Ex. Parallel plate conductors, coax cable

As a circuit element, its properties changes as a function of its cross-sectional geometry

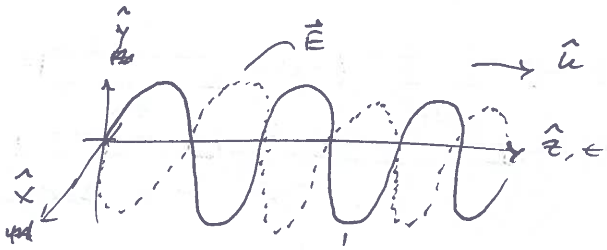


cross sectional geometry → Z_0
length → phase shift / dispersion

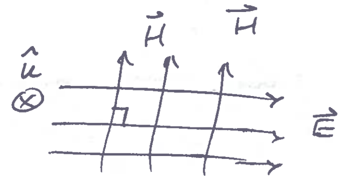
Transverse Electromagnetic Waves

One of the simpler modes of propagation

Satisfies source-free wave equation and is time harmonic $(\nabla^2 - \omega^2 \mu \epsilon)\vec{E} = 0$



E-fields and H-fields are orthogonal to each other and the direction of propagation, $\hat{u}$



Dispersion Relation: $k^2 - \omega^2 \mu \epsilon = 0$, $v_p = \frac{\omega}{k}$ Phase Velocity , $v_g = \frac{\partial \omega}{\partial k}$ Group Velocity , $\eta = \frac{|E|}{|H|}$ Wave Impedance

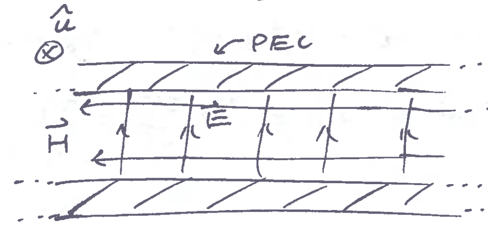
Waveguides

Structures that confine and guide electromagnetic waves

↳ Basis for Transmission Lines

Waveguides must ~~be~~ satisfy boundary conditions of Maxwell's Equations

Parallel plate Waveguide



$\vec{\nabla} \cdot \vec{D} = \rho \rightarrow \vec{E} \perp$ to metals (assuming perfect conductors)

$\vec{\nabla} \cdot \vec{B} = 0 \rightarrow \vec{H} \parallel$ to metals (assuming perfect conductors)

$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \rightarrow \vec{H}$ induced by currents and is $\parallel$

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow$ Electric field $\vec{E} \parallel$ and continuous across boundary

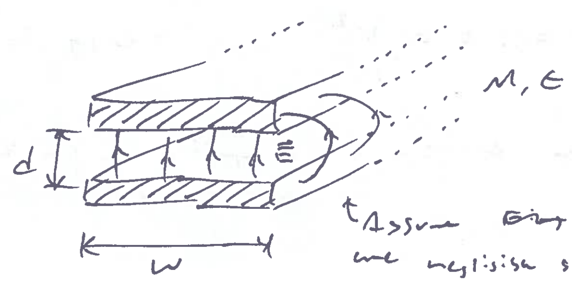
Ex. If Electric field is $\hat{x}$-directed, $\vec{E} = \hat{x} E_0 e^{-jkz} e^{j\omega t}$, $\vec{H} = -\frac{\vec{\nabla} \times \vec{E}}{j\omega\mu} = \hat{y} \frac{E_0}{\eta} e^{-jkz} e^{j\omega t}$

In a Waveguide Transmission Line, currents are able to flow for the TEM mode, so we can define a characteristic impedance based upon Voltage and Current $Z_0 = \frac{V}{I}$ ← Find I and V to find Z_0

Ignoring fringing fields

$$V = \int_0^d \vec{E} \cdot d\vec{l} = E_0 d \rightarrow \boxed{Z_0 = \sqrt{\frac{\mu}{\epsilon}} \frac{d}{W}}$$

$$I = \oint_S \vec{H} \cdot d\vec{l} = \iint \vec{J} \cdot d\vec{s} = H_0 W = \frac{E_0}{\eta} W$$

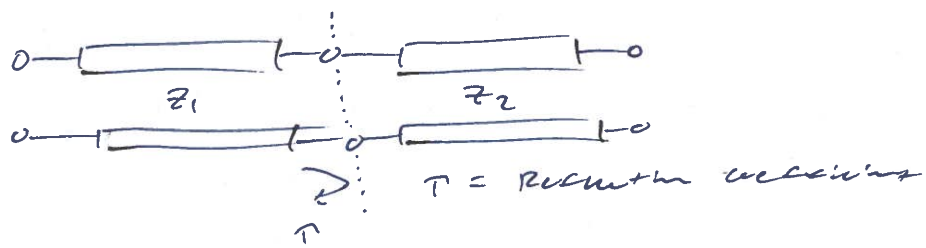


↑ Assume fringing fields are negligible since $W \gg d$

Transmission Line Theory

TEM waves will propagate through Transmission lines while maintaining Boundary conditions. Changes in Geometry (characteristic impedance) causes reflections that are due to the need to maintain boundary conditions.

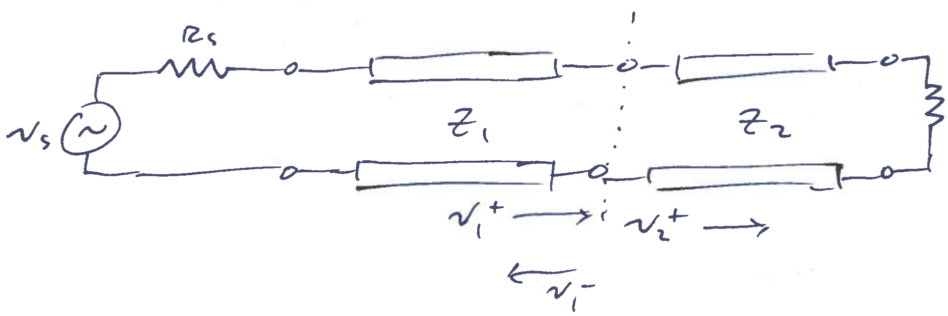
Ex. Calculate the reflection caused at the boundary between two transmission lines of different characteristic impedances.



At boundary, we have

$v_1^+ \rightarrow$, $v_1^- \leftarrow$, $\quad \Gamma = \dfrac{v_1^-}{v_1^+}$

Solve: Add source and matched load



Set $R_S = Z_2$ so that $v_2^- = 0$ (matched load, no return)

Solve with KVL \ KCL

Define $v_1^+ = v_0$, $v_1^- = \Gamma v_0$ $\rightarrow$ At boundary Voltage must be continuous $\rightarrow v_1^+ + v_1^- = v_2^+ + v_2^-$

$\hookrightarrow v_2^+ = v_1^+ + v_1^- = v_0 + \Gamma v_0$

Define $I_1^+ = \dfrac{v_1^+}{Z_1} = \dfrac{v_0}{Z_1}$, $\quad I_1^- = -\dfrac{\Gamma v_0}{Z_1}$ (↑ Different direction), $\quad I_2^+ = \dfrac{v_2^+}{Z_2}$, $\quad I_2^- = \dfrac{v_2^-}{Z_2} = 0$

KCL: $I_1^+ + I_1^- = I_2^+ + I_2^- \rightarrow I_2^+ = I_1^+ + I_1^- = \dfrac{v_0}{Z_1} - \dfrac{\Gamma v_0}{Z_1} = \dfrac{v_2^+}{Z_2} = \dfrac{v_0 + \Gamma v_0}{Z_2}$

$$\frac{Z_2}{Z_1} = \frac{1+\Gamma}{1-\Gamma} \rightarrow \boxed{\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}}$$ Reflection Coefficient

Recall that $P = IV = \dfrac{V^2}{R} \rightarrow P \sim |\Gamma|^2$ Power Reflection

Conservation of energy: $|T|^2 = 1 - |\Gamma|^2$, where T is transmission coefficient