Recitation 7: Linear Regulator + Switching Regulator

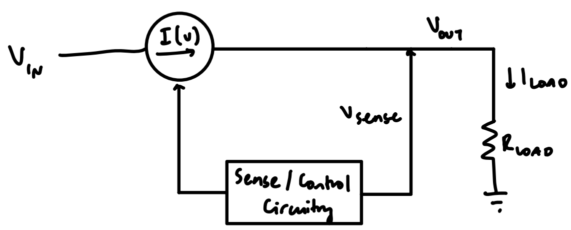
I. Voltage Regulators

- many electronic components require a stable DC voltage
- regulators maintain a constant output voltage despite variations in input voltage and load current
    - use feedback / control to correct deviations

	Linear Regulators	Switching Regulators
Control	Variable Resistance	Duty-cycle control
Loss	Heat Dissipation through resistor	Parasitics + Internal resistances of components
Efficiency	Low	High
Conversion Type	Step-Down	Step-Down, Step-Up
Regulation Mechanism	Reference and Feedback	Duty-cycle and energy-balance

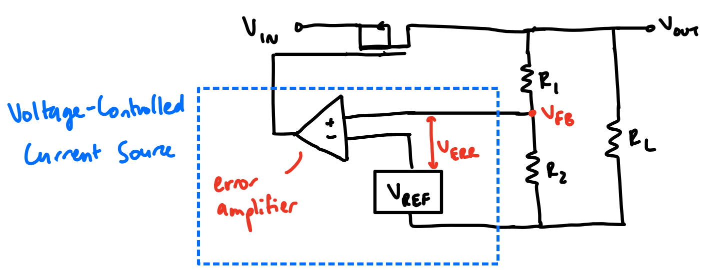
II. Linear Regulators

Block Diagram:



- control circuitry monitors the output voltage + adjusts the current source

LDO Schematic:



$$V_{EA} = A_{EA} \cdot V_{ERR}$$

- At Regulation:

$$V_{FB} = V_{REF} = \frac{R_2}{R_1 + R_2} V_{OUT}$$

- V_{OUT} too low

$V_{OUT} \downarrow$
↳ $V_{REF} > V_o$
↳ $V_{ERR} > 0$
↳ $V_{EA} \uparrow$ → device conduction increases
↳ $V_o \uparrow$

V_{OUT} too high

$V_{OUT} \uparrow$
↳ $V_{REF} < V_o$
↳ $V_{ERR} < 0$
↳ $V_{EA} \downarrow$ → device conduction decreases
↳ $V_o \downarrow$

- Dropout Voltage: Lowest differential voltage at which the circuit can regulate

$$V_{dropout} = I_o R_{DS(on)}$$

- To regulate: $V_{IN} \geq V_{OUT} + V_{dropout}$
- If $V_{IN} < V_{OUT} + V_{dropout}$, the error amplifier saturates → no regulation

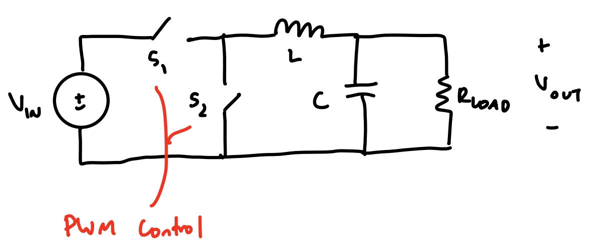
$$V_{OUT} = V_{IN} - I_o R_{DS}(on)$$

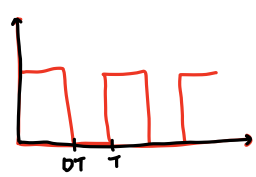
- Efficiency: $P_{loss} = (V_{IN} - V_{OUT}) I_o \rightarrow \eta = \frac{V_o}{V_{IN}}$
  - high current / high $V_{IN} - V_{OUT}$ → high power loss

III. Switching Regulators

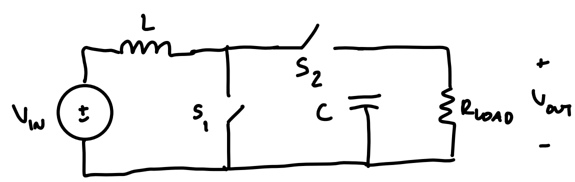
- energy flow is controlled through the ON/OFF states of transistors and transferred through inductors and capacitors

Buck Converter (Step-Down)



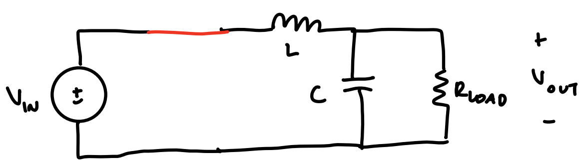


Boost Converter (Step-Up)



Buck Converter Analysis:

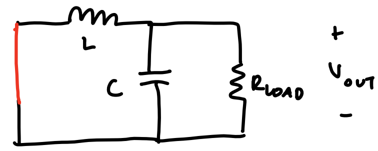
State 1: S_1 ON + S_2 OFF



$V_{L_1} = V_{IN} - V_{OUT}$

- i_L increases

State 2: S_2 ON + S_1 OFF



$V_{L_2} = -V_{OUT}$

- i_L decreases

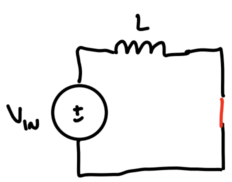
In steady-state, $\langle V_L \rangle = 0$.

$$D V_{L_1} + (1-D) V_{L_2} = 0$$
$$D(V_{IN} - V_{OUT}) + (1-D)(-V_{OUT}) = 0$$
$$DV_{IN} - DV_{OUT} - V_{OUT} + DV_{OUT} = 0$$
$$\boxed{V_{OUT} = DV_{IN}}$$

$V_{OUT} < V_{IN}$

Boost Converter Analysis:

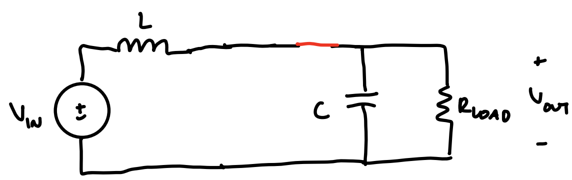
State 1: S_1 ON + S_2 OFF



$V_{L_1} = V_{IN}$

- i_L increases

State 2: S_2 ON + S_1 OFF



$V_{L_2} = V_{IN} - V_{OUT}$

In steady-state, $\langle V_L \rangle = 0$.

$$D V_{L_1} + (1-D) V_{L_2} = 0$$
$$DV_{IN} + (1-D)(V_{IN} - V_{OUT}) = 0$$
$$DV_{IN} + V_{IN} - DV_{IN} - V_{OUT} + DV_{OUT} = 0$$
$$\boxed{V_{OUT} = \frac{V_{IN}}{1-D}}$$

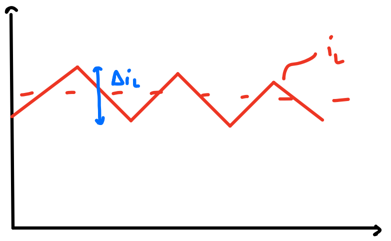
$V_{OUT} > V_{IN}$

- Efficiency: ~90-100%
  - conduction and switching losses
  - internal resistances of L and C

IV. Switching Regulator Waveforms (Buck)

Inductor $v_L = L \frac{di_L}{dt}$

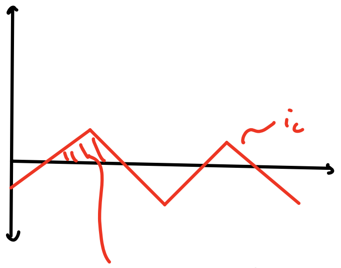
- State 1: $\frac{di_L}{dt} = \frac{V_{IN} - V_{OUT}}{L}$
- State 2: $\frac{di_L}{dt} = \frac{-V_{OUT}}{L}$



$$\Delta i_L = \frac{V_{IN} - V_{OUT}}{L} \cdot D \cdot T_s = \frac{V_{OUT}}{L}(1-D) \cdot T_s$$

switching period

Capacitor $i_c = C \frac{dv_c}{dt} = i_L - i_{OUT}$



$$\Delta Q = \frac{1}{2}\left(\frac{T_s}{2}\right)\left(\frac{\Delta i_L}{2}\right) = \frac{\Delta i_L T_s}{8}$$

$$\Delta Q = C \Delta V \rightarrow \Delta v_c = \frac{\Delta i_L T_s}{8C}$$ } output voltage ripple

- can size inductor and capacitors based on ripple constraints