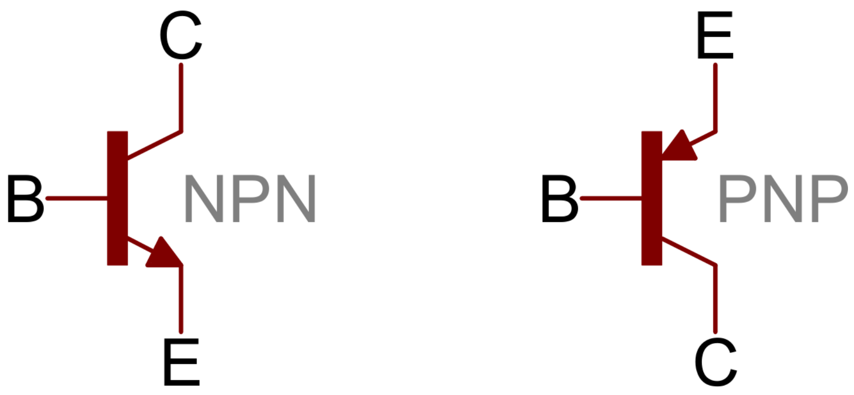
Recitation: Bipolar Junction Transistors and Small Signal Analysis

Bipolar Junction Transistor (BJT)

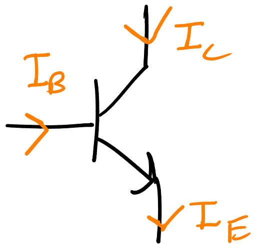


B: Base, C: Collector, E: Emitter

We'll work with NPN transistors for now. PNP are similar, but are kind of like the inverse to NPN

A quick summary of BJTs:

They act like current-controlled current sources (CCCS). Here the base is the control, and the collector/emitter are the output. Ignoring non-idealities, the current into the collector is proportional to the current into the base. Both flow out of the emitter.



$$I_C = \beta I_B \qquad I_E = I_C + I_B$$

This isn't quite enough info to build our model. Let's look at the device equation for a BJT:

$$\boxed{I_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right) \approx I_C = I_S e^{\frac{V_{BE}}{V_T}}}$$

This is a good approximation for any reasonable V_BE (V_BE > 0.1 V)

I_c: collector current
V_BE: Base-emitter voltage
I_s: saturation current, a constant that is dependent on the BJT's physical construction
V_T: Thermal voltage, a constant that equals 26mV at room temperature

It's a nonlinear equation...
Pretty annoying to work with and not terribly intuitive

Crash course in small-signal analysis: let's pretend it's linear

So our device is nonlinear, but what if our input signal is small? Maybe we can pretend it's linear? We can, and this is called small signal analysis. The technique is very reminiscent of other techniques to approximate a function as easier to work with functions, like Taylor polynomials

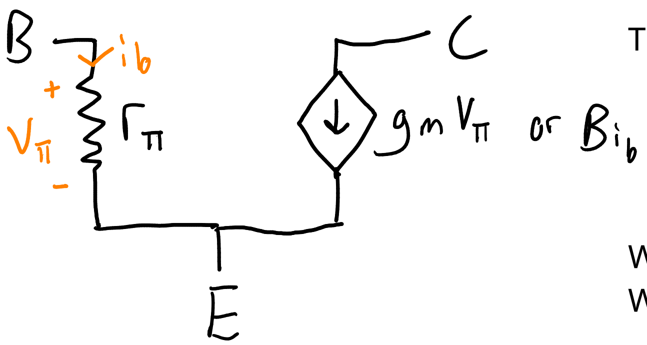
The idea here is to split our signal into a DC part and an AC part, then analyze them separately.

$$\underbrace{V_{in}}_{\text{total signal}} = \underbrace{V_{IN}}_{\text{DC}} + \underbrace{v_{in}}_{\text{AC}} \qquad \underbrace{V_{out}}_{\text{total signal}} = \underbrace{V_{OUT}}_{\text{DC}} + \underbrace{v_{out}}_{\text{AC}}$$

And since we are linearizing for the small signal (AC) part, v_out and v_in are proportional.

$$\text{Small signal: } v_{out} = \underbrace{A_v}_{\text{gain}} v_{in}$$

Let's figure out how to do this:



This is the model we'll use. (A simplified hybrid pi model)

We need to figure out what g_m and r_π are.
We will use the equation for I_c from earlier.

$$g_m = \frac{\text{collector current}}{\text{base emitter voltage}} = \frac{\partial I_C}{\partial V_{BE}} = \frac{I_S}{V_T} e^{\frac{V_{BE}}{V_T}} = \frac{I_C}{V_T}$$

$$r_\pi = \frac{\text{base emitter voltage}}{\text{base current}} = \frac{\partial V_{BE}}{\partial I_B} = \frac{\partial V_{BE}}{\partial I_C} \cdot \beta = \frac{\beta V_T}{I_C} = \frac{\beta}{g_m}$$

The big conclusions:

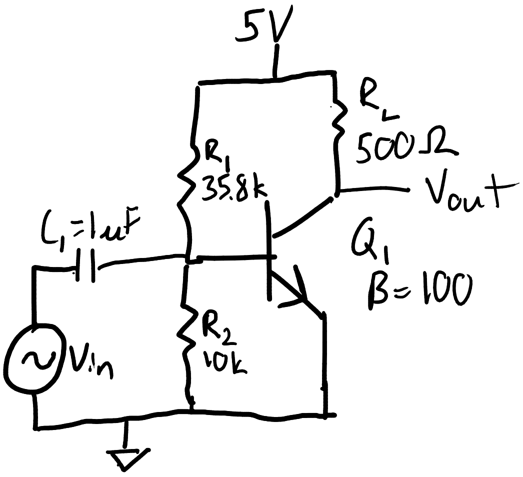
$$g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{\beta}{g_m}$$

Note: We take the derivatives in this derivation because we are interested in the small-signal parts. i.e. we are only interested effect of the perturbations on top of the DC voltage (or bias)

Here, I_C is the large signal (DC) current through the collector

An example: Common Emitter Amplifier

Let's figure out the rest of this process as we work through an example

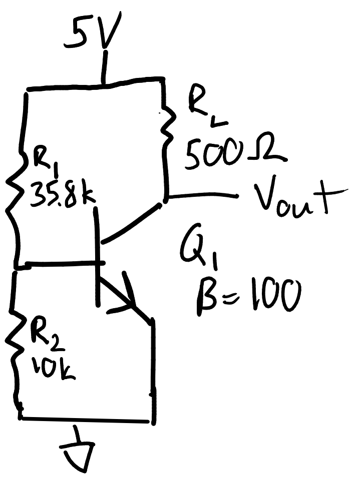


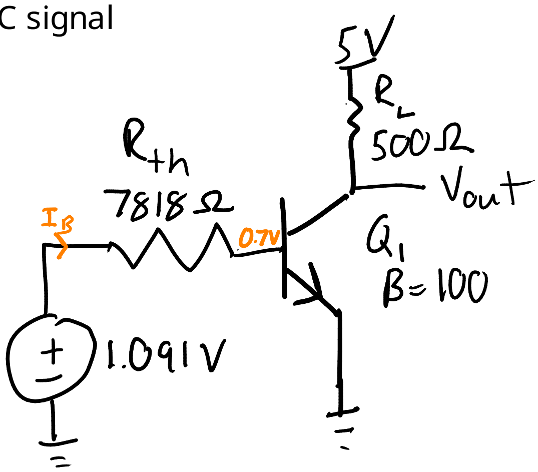
Our goal is to calculate the gain of this common-emitter amp. We will assume V_BE = 0.7V, which is often a safe assumption that is guaranteed by the device properties of the BJT.

Here C1 provides DC blocking, so that we can couple our AC signal into the amplifier while R1 and R2 set the bias

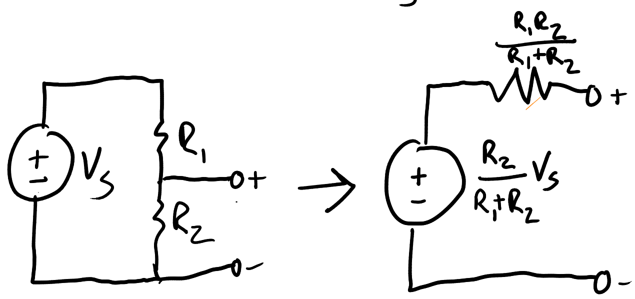
Step 1: Figure out the DC bias of the amplifier

We can ignore Vin, since that is only an AC signal





Reminder: Thevenin of a voltage divider



Now, using the assumption that V_BE=0.7V

$$I_B = \frac{1.091V - 0.7V}{7818\ \Omega} = 50\,uA$$

$$\Rightarrow I_C = \beta I_B = 5\,mA$$

Yay! We've found the bias current! Now we can calculate the small signal parameters.
Also, we observe that the output is at 2.5 V. This means we are operating in a region where our assumptions are valid.

Key Takeaway: By changing the ratio R1 and R2, we can control the bias current

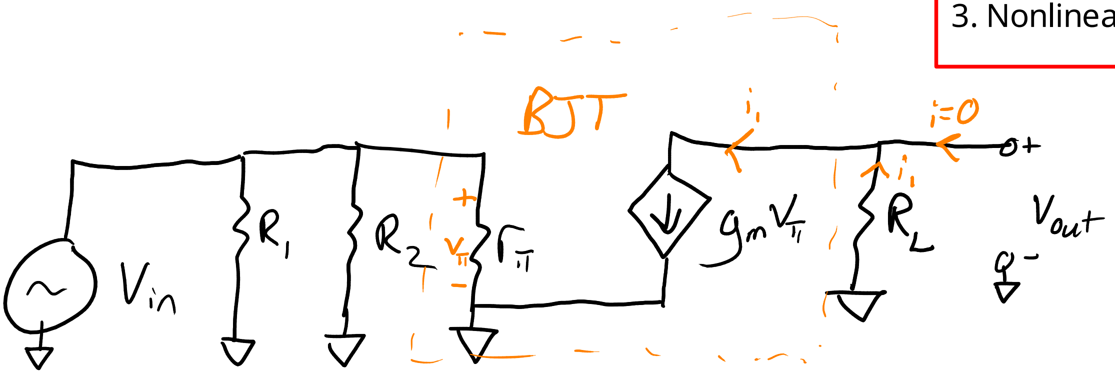
Step 2: Calculate small signal parameters

$$g_m = \frac{I_C}{V_T} = 192\,mS \qquad r_\pi = \frac{\beta}{g_m} = 520\ \Omega$$

Step 3: Draw out the small signal model

Rules for small signal models:
1. DC voltage sources become ground
2. Large capacitors can go to shorts
3. Nonlinear devices get linearized

We do these things because we don't care about the DC portion. Only the small signal, AC portion.



Step 4: Solve out to get the gain

Note: $V_\pi = V_{in} \Rightarrow i_1 = g_m V_{in}$

$V_{out} = -i_1 R_L = -g_m R_L V_{in}$

Gain: $A_v = \frac{V_{out}}{V_{in}}$

$$\boxed{A_v = -g_m R_L = -96}$$

A couple takeaways:

Our gain is increased by R_L

However, increasing R_L also decreases the DC voltage at the output.

It turns out, that if you fix the output voltage, your gain is purely a function of the supply voltage. So, a good way to think about designing these is to choose some collector current (1-10mA is a decent range for discrete transistors), then choose the other parameters to get your desired DC bias point.