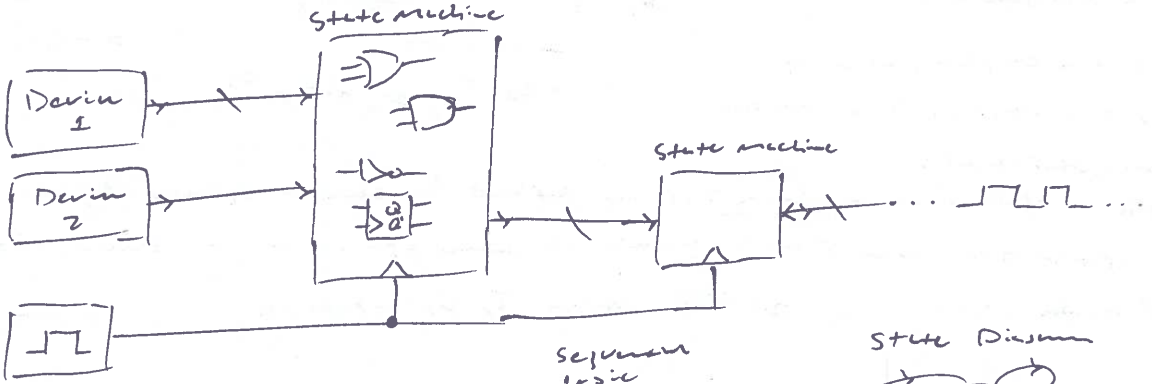
IAP 2026 PCB Course Recitation 01

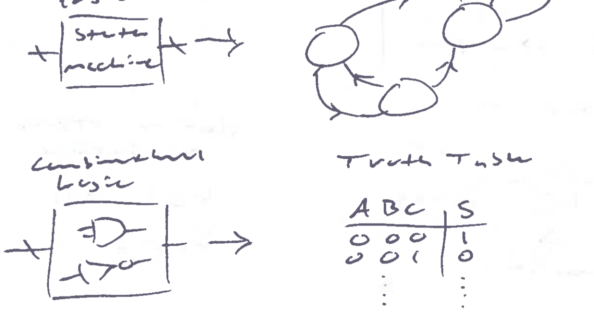
System Design

There are many ways we can represent an electronic system depending on what the device abstraction is

Digital System:

A digital system can be represented as a connection of digital blocks consisting of underlying logic gates and registers.



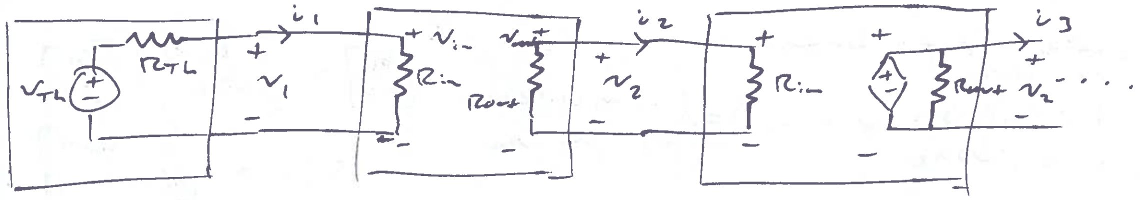


A	B	C	S
0	0	0	1
0	0	1	0
⋮			⋮

Characteristics:

- Signals are 1s and 0s only
- Instantaneous propagation
- Connections are lumped into buses
- Usually uni-directional logic flow

Analog System:

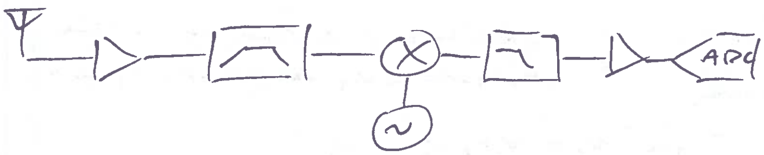


Using Circuit-Theory, we can abstract analog circuit blocks into a set of fundamental circuit elements, which can then be cascaded.

Characteristics:

- Thevenin/Norton Abstraction
- Define/solve for voltages and currents
- Use of RLC components and analysis

RF System:

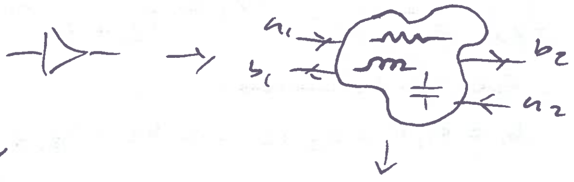


Essentially the same as the Analog System except at the sinusoidal-steady-state limit. Most components are ~~lumped~~ represented as lumped ~~as~~ S-parameter matrices.

Characteristics:

- Signals are bidirectional
- Signals represented as phasors/vectors
- Sinusoidal steady-state (~~non-steady state~~ (no Transience)

Each Device is an S-parameter matrix at each frequency, ω



$$S = \left[\cdots \right]$$

Use phasors with directionality at a particular frequency

$$v_1^+ = V_0^+ e^{j\omega t}, \quad v_1^- = V_0^- e^{j\omega t}, \text{ where } V_0^{\pm} = \underset{\substack{\uparrow \\ \text{amplitude}}}{A} e^{j\phi} \, {}^{\uparrow}\text{phase}$$

Real voltage on a node: $V_1 = \mathrm{Re}\left\{ v_1^+ + v_1^- \right\}$

S-Parameters:

Represent an RF device with bi-directional flow

$$\rightarrow \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\vec{b} = S\vec{a}$$

- a_i is a complex phasor input
- b_i is a complex phasor output

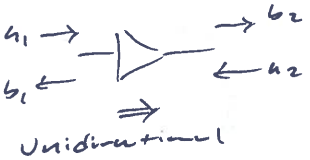
$a_i \sim Ae^{j\phi_a}$, $b_i \sim Be^{j\phi_b}$

Characteristics:

- One S-parameter matrix is typically defined for one frequency, ω.
- Input/outputs are time-harmonic → Linear operation in Fourier Domain
- To create S-matrix, define outputs $\vec{b}$ by inputs $\vec{a}$

Ideal Examples:

Amplifier:



define outputs in terms of inputs

$b_2 = Ga_1$ Where G is voltage gain.

$b_1 = 0$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ G & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad S = \begin{bmatrix} 0 & 0 \\ G & 0 \end{bmatrix}$$

↑ Valid for single frequency, ω

Filter:

define outputs in terms of inputs

$b_2 = h(\omega)a_1$ where $h(\omega) = \begin{cases} 1, & \omega_1 < \omega < \omega_2 \\ 0, & \text{otherwise} \end{cases}$

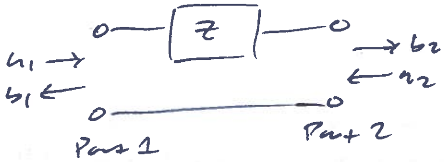
$b_1 = h(\omega)a_2$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 & h(\omega) \\ h(\omega) & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S(\omega) = \begin{bmatrix} 0 & h(\omega) \\ h(\omega) & 0 \end{bmatrix}$$

Calculating S-Parameter Matrix (Passive Circuits):

Given simple series impedance, calculate S-parameter matrix, for system with Z_0 impedance.

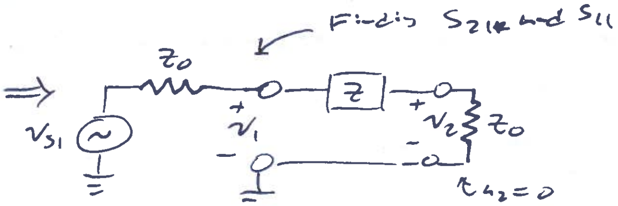


⇒ $a_1 = v_1^+$, $a_2 = v_2^+$

⇒ $b_1 = v_1^-$, $b_2 = v_2^-$

Represent system with Voltages\currents

⇒



Calculate one port at a time by terminating all other ports.

Calculation:

Given that Port 1 has a source and Port 2 is terminated,

$v_1^+ = \frac{1}{2}v_{S1}$, $v_2^+ = 0$ → $v_1 = v_1^+ + v_1^- = \frac{1}{2}v_{S1} + v_1^-$ and $v_2 = v_2^+ + v_2^- = v_2^-$

Use node method to find voltages v_1 and v_2,

$$v_1 = \frac{Z + Z_0}{2Z_0 + Z}v_{S1}, \quad v_2 = \frac{Z_0}{2Z_0 + Z}v_{S1}$$

Solve for S-Parameters:

Recall: $b_1 = S_{11}a_1 + S_{12}a_2$ and $b_2 = S_{21}a_1 + S_{22}a_2$ →

$$S_{11} = \frac{b_1}{a_1}\Big|_{a_2=0}, \quad S_{12} = \frac{b_1}{a_2}\Big|_{a_1=0}$$

$$S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0}, \quad S_{22} = \frac{b_2}{a_2}\Big|_{a_1=0}$$

$$S_{11} = \frac{b_1}{a_1}\Big|_{a_2=0} = \frac{v_1^-}{v_1^+}\Big|_{v_2^+=0} = \frac{v_1 - \frac{1}{2}v_{S1}}{\frac{1}{2}v_{S1}} = \frac{Z}{2Z_0 + Z}$$

$$S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0} = \frac{v_2^-}{v_1^+}\Big|_{v_2^+=0} = \frac{v_2}{\frac{1}{2}v_{S1}} = \frac{2Z_0}{2Z_0 + Z}$$

Since this circuit is passive → Reciprocal S-Parameter matrix is Symmetric, $S = S^T$

$S_{11} = S_{22}$

$S_{21} = S_{12}$

$$\rightarrow S = \begin{bmatrix} \frac{Z}{2Z_0+Z} & \frac{2Z_0}{2Z_0+Z} \\ \frac{2Z_0}{2Z_0+Z} & \frac{Z}{2Z_0+Z} \end{bmatrix} \checkmark$$

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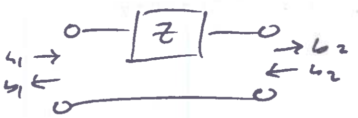


$$\Big\} = S = \begin{bmatrix} \frac{Z}{2Z_0+Z} & \frac{2Z_0}{2Z_0+Z} \\ \frac{2Z_0}{2Z_0+Z} & \frac{Z}{2Z_0+Z} \end{bmatrix} \rightarrow$$

$S_{21} = S_{12} = \frac{2Z_0}{2Z_0+Z}$ Power flowing through the device, input → output

$S_{11} = S_{22} = \frac{Z}{2Z_0+Z}$ Power reflected by the device at its inputs

Double check:

Set Z to be a capacitor (DC-Block), $Z = \frac{1}{j\omega C}$

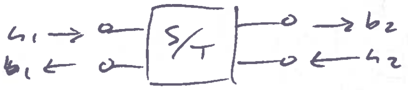
- As Frequency increases, $\omega \uparrow$, $Z \downarrow$, $S_{11} \rightarrow 0$, $S_{21} \rightarrow 1$ $\Rightarrow$ All power passes through capacitor
- As Frequency decreases, $\omega \downarrow$, $Z \uparrow$, $S_{11} \rightarrow 1$, $S_{21} \rightarrow 0$ $\Rightarrow$ All power is reflected back

Cascading S-Parameters:

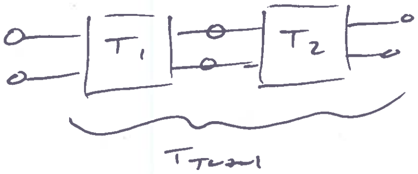
Convert S-matrices into their equivalent T-Matrices: $T_{11} = S_{12} - \frac{S_{11}S_{22}}{S_{21}}$, $T_{12} = \frac{S_{11}}{S_{21}}$

$T_{21} = -\frac{S_{22}}{S_{21}}$, $T_{22} = 1/S_{21}$

$$\underbrace{\begin{bmatrix} b_1 \\ a_1 \end{bmatrix}}_{\text{Port 1}} = \underbrace{\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}}_{T} \underbrace{\begin{bmatrix} a_2 \\ b_2 \end{bmatrix}}_{\text{Port 2}}$$



Cascade T-matrix by matrix multiplication:



$T_{Total} = T_1 T_2$

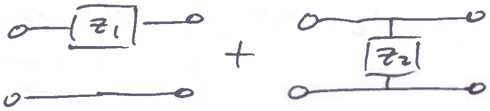
∗ Also can use ABCD matrix equivalently.

This kind of analysis is what is used in advanced RF/microwave simulation software.

Example:

RC Lowpass filter

$\rightarrow$



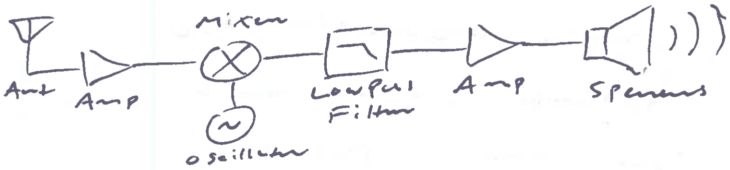
Cascade S-matrix

$\rightarrow$ $T_1 T_2$

multiply T-matrix

FM Radio Receiver System (How it works):

Simplified FM Radio Receiver Block Diagram

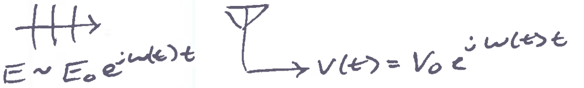


- We could represent each component as an S-matrix and cascade using T-matrix.
- But since this is an ideal system we can approximate that it is Unidirectional and use simple functions,

$$x \rightarrow \boxed{f} \rightarrow f(x)$$

Define abstracted component functions:

Antenna (source)



$E \sim E_0 e^{j\omega(t)t}$ $\qquad V(t) = V_0 e^{j\omega(t)t}$

Amplifier

$V_0 e^{j\omega(t)t} \rightarrow \; \rightarrow G V_0 e^{j\omega(t)t}$

Oscillator (source)

$\rightarrow V_0 e^{j\omega_0 t}$

Lowpass Filter

$$V_0 e^{j\omega t} \rightarrow \; \rightarrow \begin{cases} V_0 e^{j\omega t}, & \omega < \omega_c \\ 0, & \omega > \omega_c \end{cases}$$

Mixer

$\rightarrow f(x,y) = \mathrm{Re}\{x\}\mathrm{Re}\{y\}$

$x \sim e^{j\omega t}$

$y \sim e^{j\omega t}$

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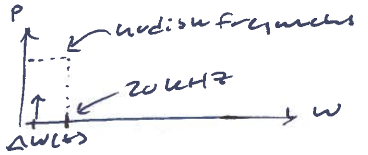
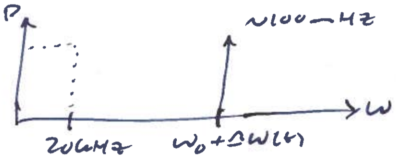
FM Radio consists of a high frequency carrier signal (~100 MHz) that has its frequency modulated slightly over time with an audio signal

Audio Signal ~ $\cos(\Delta\omega(t)t)$



FM Signal ~ $\cos(\omega_0 t + \Delta\omega(t)t)$



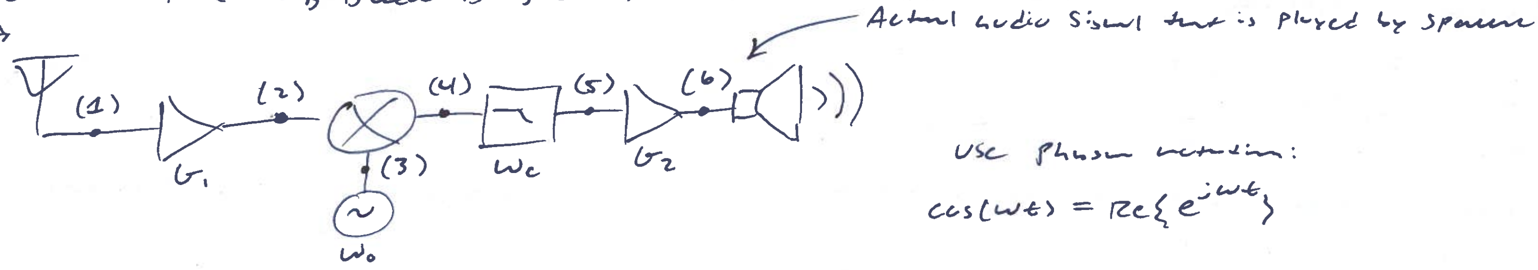
$\omega_0 \sim 100\text{ MHz}$

$\Delta\omega(t) \sim 10\text{ kHz}$

$\omega_0 \gg \Delta\omega(t)$

The FM Radio Receiver needs to downconvert the FM signal by removing the carrier frequency, ω_0, and bringing the audio signal ~~[illegible]~~ back to the audible range.

Demonstration Using Block Diagram:



Use phasor notation:

$\cos(\omega t) = \mathrm{Re}\{e^{j\omega t}\}$

(1) $V(t) = V_0 \cos((\omega_0 + \Delta\omega(t))t)$

(2) $G_1 V_0 \cos((\omega_0 + \Delta\omega(t))t)$

(3) $V_1 \cos(\omega_0 t)$

(4) $G_1 V_0 \cos((\omega_0+\Delta\omega(t))t) \cdot V_1 \cos(\omega_0 t) = \frac{1}{2} G_1 V_0 V_1 \left(\underbrace{\cos((2\omega_0+\Delta\omega(t))t)}_{\text{High frequency}} + \underbrace{\cos(\Delta\omega(t)t)}_{\text{Audio Signal}}\right)$

(5) $\frac{1}{2} G_1 V_0 V_1 \cos(\Delta\omega(t)t)$

(6) $\frac{1}{2} G_2 G_1 V_0 V_1 \cos(\Delta\omega(t)t)$ ✓ Done!

↑ Audio recovered

Now that we have designed an FM Radio System architecture, we must now build its individual components separately, and attempt to optimize them to be as close to their ideal forms as possible.